

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering
ECE 2040

Lab #3: AC Circuit Analysis and Frequency Response of First and Second-Order Circuits

This lab is due one week from the day it is assigned in class. You may work in groups of two, however each student is expected to have their own answer sheet.

All answers should be recorded on the answer sheet found in the appendix. Failure to do so will result in a loss of a letter grade on this lab. Any graphs you print should be appended to the answer sheet and turned in at the same time as the answer sheet.

GOAL

To gain experience measuring the frequency response of first and second-order AC circuits.

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EQUIPMENT NEEDED

- Two pieces of semi-log paper (Can be found on Page 13 or on the internet)
- 1k Ω resistor
- 51 Ω resistor
- 3.3mH inductor
- 0.22 μ F capacitor
- TL082 op-amp
- Breadboard
- Jumper wires
- NI myDAQ

BACKGROUND

Frequency Response of Circuits: A circuit is a system with input $x(t)$ and output $y(t)$. Usually, the input is a voltage source, where $x(t) = v_s(t)$, but a current source may also be used, $x(t) = i_s(t)$. The output is identified as a voltage or current in the circuit.

Phasor representation is used in AC circuit analysis to understand the steady-state response of sinusoidal inputs signals.

The frequency response of a circuit is the behavior of the system as it processes sinusoids of different frequencies. The frequency response function, $H(\omega)$, is found mathematically using the impedance method or through experimental measurements by comparing the amplitude and phases of the input and output waveforms.

Experimental Determination of Frequency Response: Examine Figure 1, which shows the input-output behavior of a circuit with frequency response function $H(\omega)$ with sinusoidal input amplitude A_i and frequency ω_1 rad/sec. The steady-state output is also a sinusoid with the same frequency, ω_1 , but with a scaled amplitude, A_o , and phase lag ϕ .

The frequency response plot is:

- 1) A plot of the amplitude scaling $|H(\omega)| = A_o/A_i$ versus the frequency ω .
- 2) A plot of the phase lag ϕ versus frequency ω .

The data is commonly plotted using log scales (due to amplifier and filter design) for magnitude and frequency, resulting in a Bode plot representation of the frequency response. Often times the magnitude will also be converted to decibels (dB) which can be done by setting $|H(\omega)_{db}| = 20\log(|H(\omega)|)$. This is done since sound pressure level is measured in dB, which means to our ears a noise that sounds 2x louder is in fact 10x as loud (hence the 20log scaling).

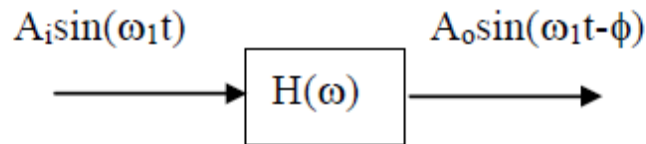


Figure 1: Input-Output Behavior of a Linear Circuit in Steady State.

The output amplitude, A_o , and the output phase, ϕ , depend on $H(\omega)$ as follows:

$$A_o = A_i |H(\omega_1)| \quad \text{and} \quad \phi = \angle H(\omega_1)$$

To determine $H(\omega)$ from experimental measurements of a circuit, find the steady-state sinusoidal response to a wide range of input frequencies. From each sinusoidal response, measure A_o and ϕ . Fill in a table such as the one in Figure 2 for each input frequency.

ω_1	$\phi = \angle H(\omega_1)$	$ H(\omega_1) = \frac{A_o}{A_i}$	$20 \log H(\omega_1) $

Figure 2. Table for $H(\omega)$

Consider, for example, the response, $y(t)$, of a system with input $x(t)=\sin(3\pi t)$ shown in Figure 3. For this input, $\omega_1=3\pi$, and $A_i = 1$. For the signal in steady-state, measure A_o and ΔT , the time lag. The phase lag in degrees is calculated as:

$$\phi = -\frac{\Delta T}{T} * 360^\circ$$

where T is the period of the signal.

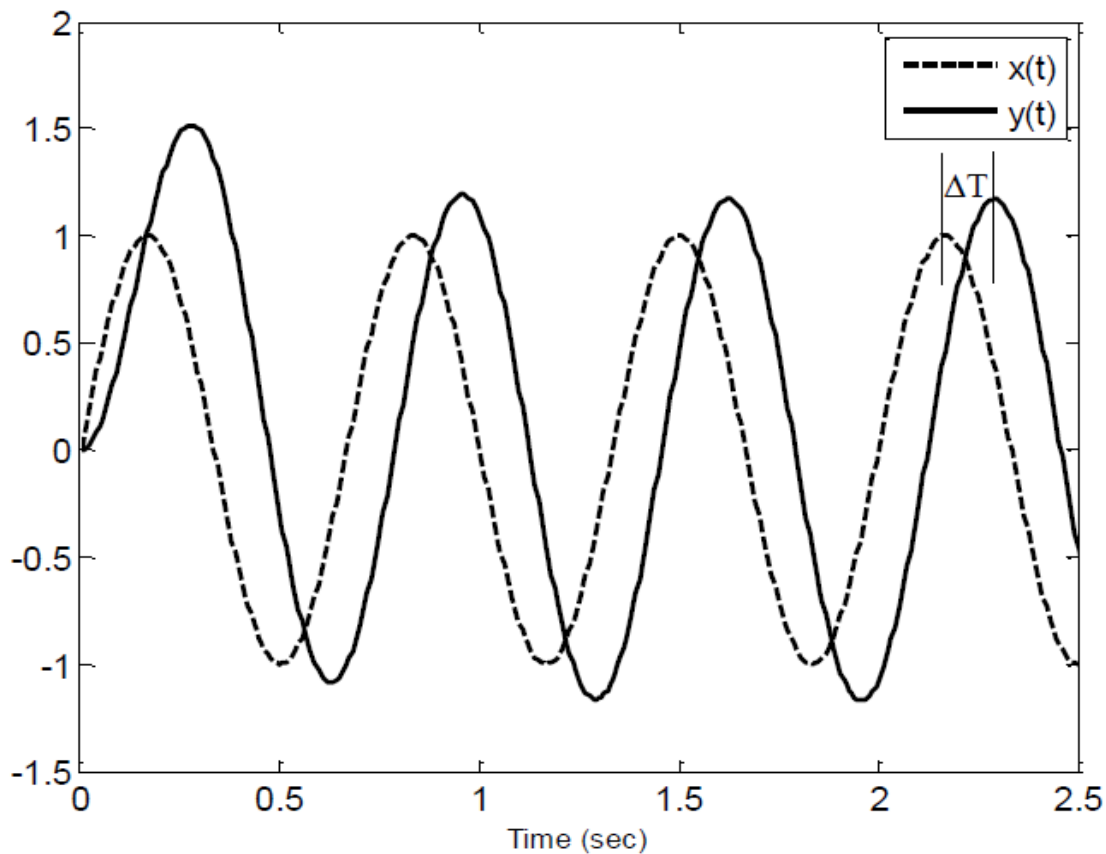


Figure 3. Plot of $x(t)$ and $y(t)$ in the time domain

The period $T = 2\pi/\omega_1 = 0.67$. From the plot, $\Delta T = 0.12$ s and $A_o = 1.17$. So $|H(\omega_1)| = A_o/A_i = 1.17$ and $\phi = -0.12(360)/T = -64.7^\circ$.

ω_1	$\phi = \angle H(\omega_1)$	$ H(\omega_1) = \frac{A_o}{A_i}$	$20 \log H(\omega_1) $
3π	-64.7°	1.17	1.36 dB

Select a wide range of values of input frequencies that fully characterize the frequency response.

Once the data is collected, plot all the points on semi-log paper to obtain the Bode plot of the system (this will look like Figure 4).

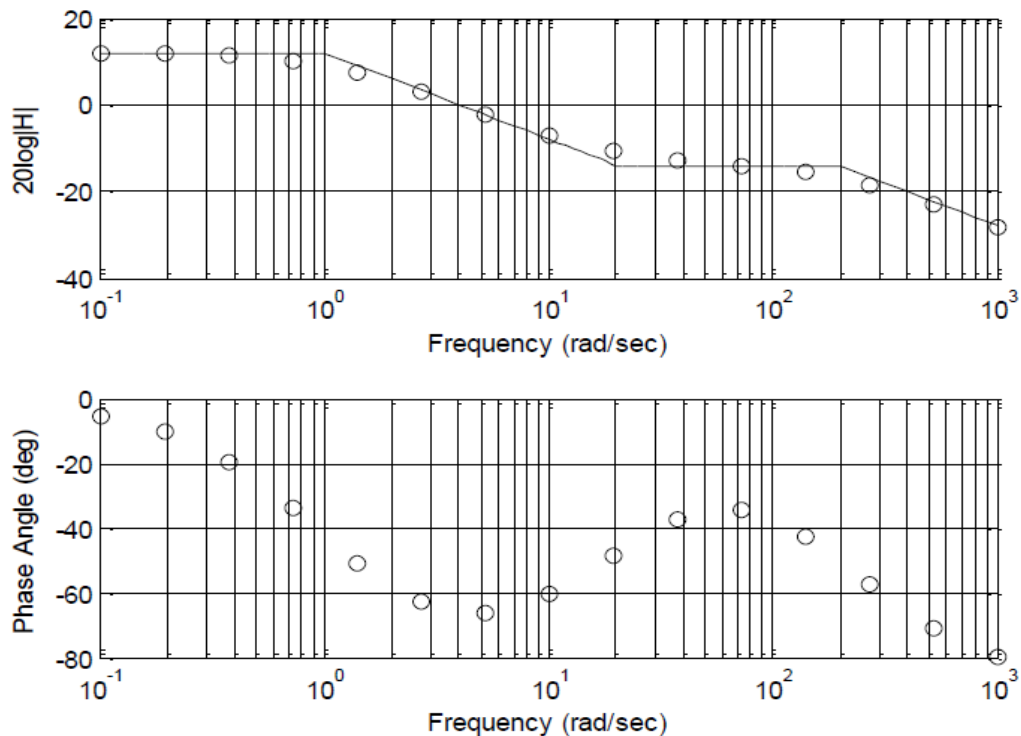


Figure 4. Frequency response of a system. Magnitude plot is seen on top and is plotted in decibel scale from 0.1Hz to 1000Hz. Phase plot is seen on bottom and is plotted in degrees from 0.1Hz to 1000Hz.

Bandwidth: When a magnitude plot starts flat at low frequencies, then drops off as the frequency increases, then the bandwidth is defined as the frequency at which the magnitude is equal to $\frac{\sqrt{2}}{2}$ (~ 0.707) of the DC value, that is $|H(\omega_b)| = 0.707 |H(0)|$. On the Bode plot, the log magnitude, $20\log |H|$, is 3dB below that of the DC value, $20\log |H(0)|$.

Resistors: The resistance of physical resistors is denoted by four color bands on the resistor. The color code for bands 1-3 is shown in Figure 5

Color	Value	1 st band and 2 nd band give the first two significant numbers of the resistance 3 rd band gives the base 10 multiplier, $\times 10^n$ 4 th band gives the tolerance (silver is $\pm 10\%$ and gold is $\pm 5\%$)
Black	0	
Brown	1	
Red	2	
Orange	3	
Yellow	4	
Green	5	
Blue	6	
Purple	7	
Grey	8	
White	9	

A resistor with bands (yellow, red, orange, silver) is a $42,000 \Omega$ resistor with a tolerance of $\pm 10\%$.



Figure 5. Band Resistor Color

Code

National Instruments myDAQ Input/Output Interfaces: The myDAQ has an interface on one side with +15 V, -15 V, Analog Output (AO), Analog Input (AI), and digital input/outputs. The part of the interface that will be used in this experiment is shown below.

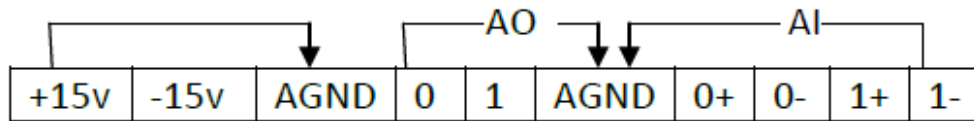


Figure 6: myDAQ Terminals (+/- 15V, AO, AI) Used in This Lab

The scope, short for oscilloscope, is used to plot the inputs in (AI) Channels 0 and 1, that is, the potential between AI0- and AI0+ and between AI1- and AI1+. These values are plotted with respect to time.

Op Amp Buffers Circuits: Since op amps have their own power source, they can be used to build a unity gain buffer circuit whose purpose is to boost the power without changing the voltage waveform. This would result in a higher current since $P = IV$ is boosted with voltage remaining constant. The myDAQ analog output, AO, channels are limited to 2mA, so the AO channels can be passed through a unity gain op amp in order to deliver higher current levels. The pin diagram and schematic of an integrated chip (IC) containing an op amp is shown in the Figure 7. The IC pin number 1 is the one closest to the circle stamped into the top of the IC.

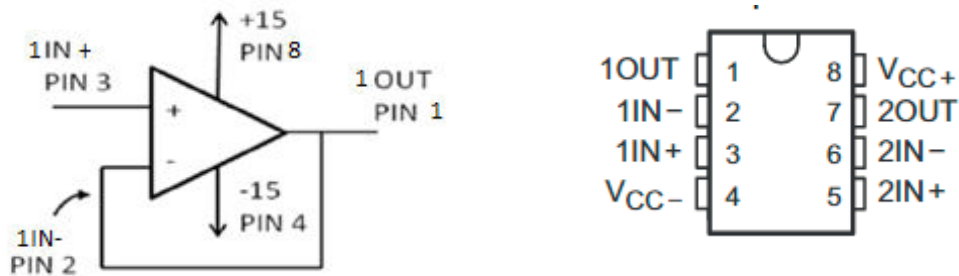


Figure 7. TL082 Op Amp Circuit Symbol and Pin Assignments(left) and TL082 IC Pinout (right)

PRE-LAB ASSIGNMENT

1. Every Student needs to download the NI ELVISmx software, install it on your laptop, and bring your laptop to class. Make sure that the NI ELVIS Instrument Launcher software runs. *This process takes about an hour to do.* Failure to do this may result in not being able to perform the laboratory exercise. The software can be found on the National Instruments website at: <http://joule.ni.com/nidu/cds/view/p/id/2157/lang/en>.

For students using a MAC, you will need to install the software on a Windows partition and run it from there. The experiment will be done in pairs, so if you cannot get this software to run on your machine, then make sure that you arrange to have a partner who can run it.

2. Read the Background section of this laboratory exercise.
3. Print out a copy of the answer sheet appended at the end of this document and bring it to lab.

LAB PROCEDURES

1. Setup

- a. Connect the myDAQ to your computer using the USB connector.
- b. Start up the NI ELVISmx Instrument Launcher software.
- c. Start up the function generator FGEN and the SCOPE by clicking on their icons.

2. Using an Op Amp Buffer Circuit

- a. Connect a 1k Ω resistor to the function generator (AO wires) and to Channel 0 of the scope (AI wires) as shown in the Figure 8.

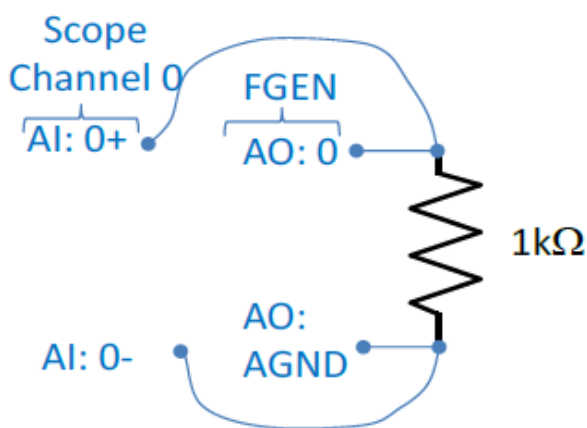


Figure 8. Resistor Wired to Both the myDAQ's Function Generator (AO: 0/AGND) and Oscilloscope (AI: 0+/-)

- b. Apply a 1 V peak to peak sinusoidal signal across the resistor using FGEN, and view the sine wave on the SCOPE. The corresponding instrument settings are on the next page.

- Function Generator
 - Produce a sine wave voltage with frequency $f = 500 \text{ Hz}$ ($=1000\pi \text{ rad/sec}$) and 1 V peak-to-peak with a dc offset of 0 V. That is, $v_s = 0.5 \sin((1000\pi)t)$.
 - Oscilloscope
 - Set the trigger type to edge, the level to 0.1 V, and the horizontal position to 10%
 - Set scale for Volts/Div to 1V
 - Set the Time/Div setting to 2ms
- c. Click Run on both the function generator and the oscilloscope.
- d. **[1]** Slowly increase the amplitude of the sine wave from FGEN until the voltage signal starts clipping (ie, the peaks flatten). Note the amplitude of the clipping. Sketch the shape of the waveform and explain what is happening (hint: read the background section on buffer circuits).
- e. Insert the buffer op amp circuit between the myDAQ output AO channel 0 and the resistor as shown in the schematic.
- f. Connect pins 4 and 7 of the op amp IC to the myDAQ +15v and -15v power supply (use the op amp pin diagram in the background section as a guide).

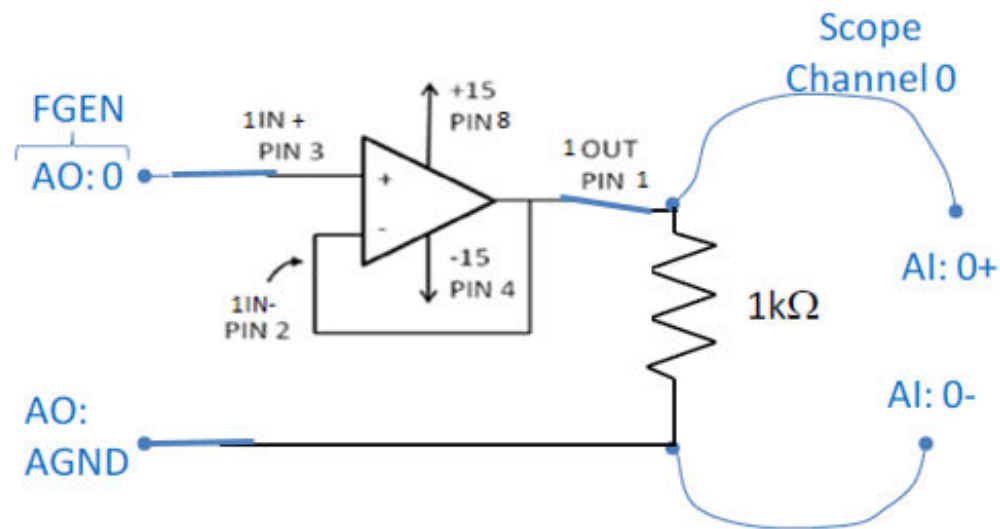


Figure 9. Resistor Wired With Buffer Op Amp(TL082) Circuit

- g. **[2]** Increase the amplitude of the sine wave up to 10 volts peak to peak. Does the signal clip? If not why not? Click “Stop” on the FGEN when finished.

3. Measuring Sinusoidal Steady-State Behavior:

- a. Use the breadboard to build the circuit in Figure 11.

Voltage source provided by the function generator, FGEN, passed through the buffer circuit

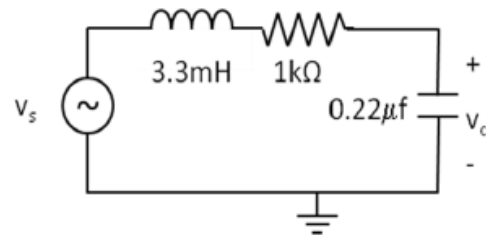


Figure 10: RLC Circuit Schematic with an AC Voltage Source (V_s)

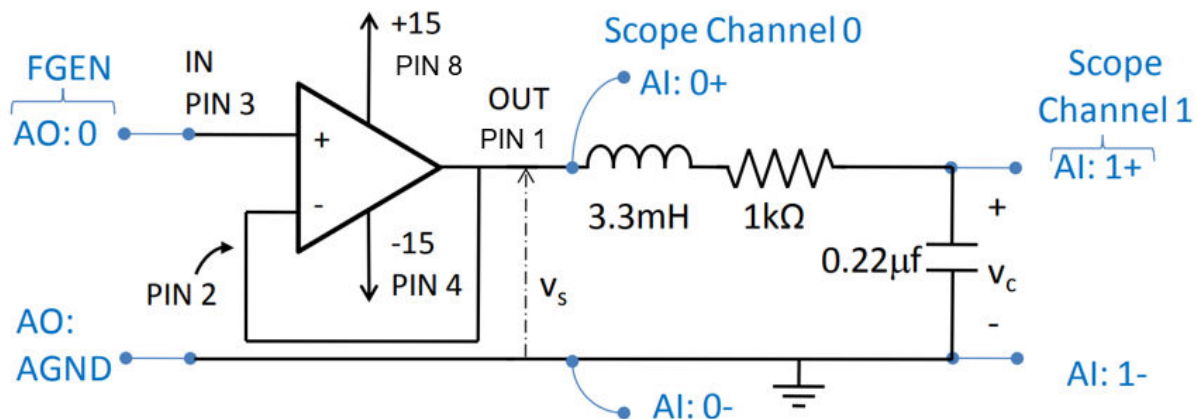


Figure 11: RLC Circuit Showing Unity Gain Op Amp and myDAQ Connections

- b. **[3]** Calculate the steady-state amplitude and phase lag (in degrees) of V_c to an input $V_s = \sin(2\pi f)$ where $f = 500$ Hz ($\omega = 2\pi f$).
- c. To measure the steady-state output V_c sine wave of your circuit to an input of $v_s = \sin(2\pi f)$ for $f = 500$ Hz follow these steps:
- Function Generator
 - Set the frequency on FGEN to 500 Hz
 - Set the amplitude to 2 Vpp (that is, peak to peak)
 - Oscilloscope
 - Enable Channel 1 and set to AI 1
 - Time/Div = 500 μ s
 - Scale Volt/Div = 1v on both channels
 - Vertical position = 0v (ie, no DC bias)
 - Turn on the cursors (box is in the bottom left of the SCOPE) and set C1 to CH0 and C2 to CH1

- d. **[4]** Measure the amplitude and lag of V_c . Drag the line for cursor 2 (C2) from the left axis until the “+” sign is aligned with the peak of the CH 1 (blue) sine wave. Read the corresponding voltage for C2 (below the plot). This is the amplitude.
- e. **[5]** Drag cursor 1 (C1) from the left axis until the “+” sign is aligned with the peak of the CH 0 (green) sine wave. Read the time difference between the input sine wave peak (C1) and output sine wave peak (C2), that is, the dT value under the plots.
- f. **[6]** Compare the experimental results to the analytical prediction of the amplitude and phase lag you computed.

4. Frequency Response Data Table:

- a) **[7]** To determine the frequency response experimentally using the method described in the background section, you must take amplitude and phase lag measurements at a number of input frequencies. Start by recording the ΔT and amplitude measurements for $f = 500$ Hz measured in Step 10 into the table below. Note that $|H(f)| = A_o/A_i = A_o$ when $A_i = 1$. Repeat the procedure done for $f = 500$ Hz for all of the frequencies in the table on the answer sheet.
- b) Compute the log magnitude $20\log |H(f)|$ and phase lag ϕ at each frequency to complete the table.
- c) **[8]** Transfer these data points to semi-log paper (which can be found in Appendix B). Draw a straight-line approximation for the Bode plot on the semi-log paper. You should have two plots: one of magnitude vs. frequency and the other of phase vs. frequency. Make sure to label your axes. Under problem 8 note the difference between the actual data and the approximation.

5. Responses in the Time Domain and in the Frequency Domain:

- a. **[9]** With the same RLC circuit used in the previous part, slowly increase the frequency of the input signal in FGEN (change the time/div scale on the SCOPE as needed to see the sine wave clearly). This procedure is called a *sine sweep*. Describe the effect on the amplitude and phase lag of the output signal behavior as the frequency is increased.
- b. **[10]** Find the bandwidth, that is, the frequency at which the output amplitude = 0.707 times its low frequency amplitude.

6. Bode Plot: In this section we will be using the Bode Plot tool. The Bode Plot tool sends a sweep of frequencies through a circuit and plots the frequency response.

- a. Close FGEN and the Oscilloscope and open the BODE window.
- b. Choose 15 steps per decade and run the Bode analyzer.

- c. **[11]** Take a screenshot of the graph and append it to the back of the lab report (make sure to label the graph). Under problem 11 in the answer sheet make a note of how this graph compares to the graph you sketched by hand.
- 7. Low Damping:** In this section you will see the effect of having a small R value, which results in low damping in the time domain and a high resonance peak.
- a. Replace the $1\text{k}\Omega$ resistor with a 50Ω resistor and repeat the first two steps under the “Bode Plot” section.
 - b. **[12]** Take a screenshot of the graph and append it to the back of the lab report (make sure to label the graph). Under problem 12 in the answer sheet make a note of how this graph compares to the graph you got in problem 11
 - c. **[13]** Determine the approximate resonant frequency, where the maximum output amplitude occurs.
 - d. **[14]** Determine the bandwidth (3dB below the low frequency value). This value should be measured on the right side of the resonance peak.
 - e. **[15]** Compare your bandwidths from the two circuits/graphs. Which one has a better (larger) bandwidth? What possible downsides could there be to having this larger bandwidth (look at your graphs)?

APPENDIX A: Answer Sheet

1.

2.

3.

4.

5.

6.

7.

Fill out this table

f(Hz)	ΔT	$\phi = \angle H(f)$	$H(f) = A_o/A_i$	$20 \log H(f)$
100				
200				
500				
1000				
2000				
6000				
10,000				

8.

9.

10.

11.

12.

13.

14.

15.

APPENDIX B: Semi Log Paper Sheets