

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering
ECE 3084 Spring 2019

Lab #2: Frequency-Domain Characteristics of a Guitar String

This lab is due one week from the day it is assigned in class. You may work in groups of two and share a checkoff sheet, however you are expected to print your own checkoff sheet and bring it to class.

GOAL

The goal of this experiment is to learn about the relationship between time domain and frequency domain signal characteristics, harmonic behavior, impulse response, Fourier transforms, decoding frequency response characteristics from experimental data, and using a digital acquisition board (myDAQ) to take measurements and analyze their characteristics.

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EQUIPMENT NEEDED

- myDAQ
- Guitar string setup

PRE-LAB

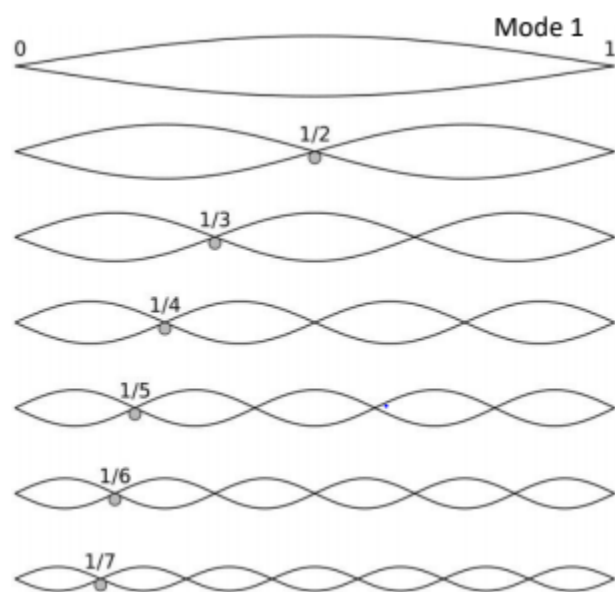
1. Download and install the NI ELVISmx (Version 4.5) software from <http://www.ni.com/download/ni-elvismx-4.5/4316/en/> (Mac users must use the Windows partition on their system or partner with someone who can run Windows. You will not need to download or run LabVIEW.) **YOU MUST INSTALL THE SOFTWARE BEFORE COMING TO LAB!!!**
2. Watch the video that describes the use of the myDAQ at <http://www.ni.com/mydaq/support>
3. Watch this video on measuring the time-domain response of a vibrating guitar string: http://www-srv2.ece.gatech.edu/academic/Fall12/TESSAL/TESSAL_GuitarImpResp.html
4. Watch this video on measuring the frequency-domain response of a vibrating guitar spring using a small speaker to create a sinusoidal input (this is fairly tricky to do, so you won't be doing it lab, but you should watch the video): http://www-srv2.ece.gatech.edu/academic/Fall12/TESSAL/TESSAL_GuitarFreqResp.html

BACKGROUND

Examples of transducers – guitar pickups: A transducer is a device that converts a physical quantity into an electrical signal that can be measured. A guitar pickup is a transducer that converts vibration of a guitar string into a voltage signal. The pickup has a permanent magnet wrapped with a copper coil that creates a magnetic field. A metal string vibrating within that magnetic field induces an electrical signal that is proportional to the velocity of the vibration of the string at the point of measurement.

Dynamic Signal (Spectrum) Analyzer (DSA): A spectrum analyzer measures a signal from a transducer and computes the Fourier Transform of the signal, which is displayed on a graph of magnitude versus frequency. Internally, the spectrum analyzer uses the FFT of the sampled signal to compute an approximation to the exact Fourier Transform. Also, DSAs are often programmed to compute the Fourier Transform (using an FFT) on a window of data. This means that the DSA takes a window of data and computes the frequency spectrum on that window of data, displays it, and then takes a new window of data to repeat the process.

Vibrating strings: A string attached at both ends has a dynamic response when excited externally, such as being plucked like a guitar string or hit like a piano string. Since neither end of the string can move, the string vibrates following specific modal shapes with the amount of deflection dependent on the position along the string. For example, plucking the string in the middle will excite the string to vibrate in the shape of Mode 1, shown below.



The first seven mode shapes, counting from mode 1 at the top. The marked points are nodes where the string does not move for that mode. The Mode 2 vibrates at twice the frequency of the first mode. Image from Vibrating String, http://en.wikipedia.org/wiki/Vibrating_string

To determine the frequency (in Hertz) of each mode, use the expression

$$f = \frac{k}{2L} \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string, L is the string length, μ is the density of the string (mass per unit length), and k is the mode number. The frequency can be tuned by increasing the tension or changing the length. In stringed musical instruments, such as a guitar, the tuner is used to tighten the string, i.e., change the tension. Frets are bars placed across the strings that effectively change the length of the string.

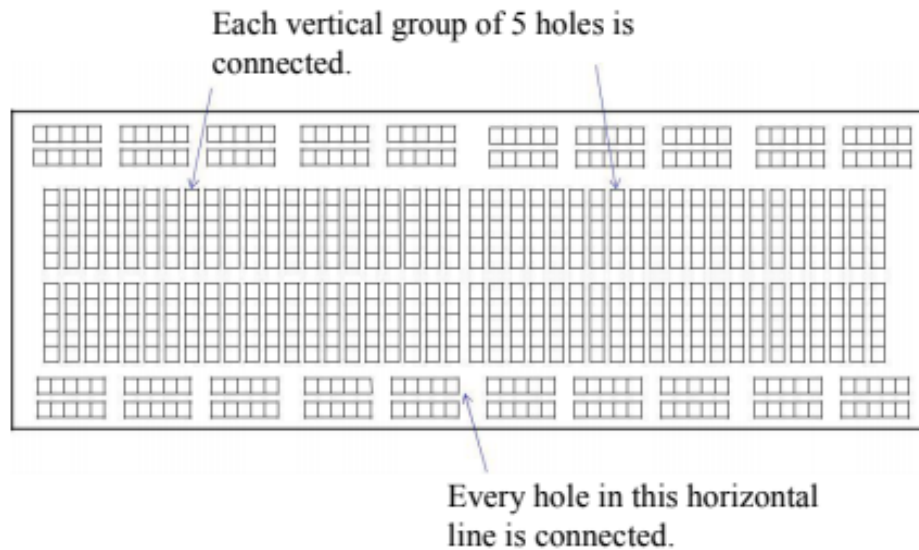
The fundamental frequency is the frequency of the first mode, $k=1$. Harmonics are the higher frequencies, so the second harmonic is the mode shape and frequency corresponding to $k=2$.

Free and forced response of a string: When you pluck a guitar string, you are momentarily giving it an initial condition on the displacement. The resulting response is a zero input response (also known as the free response) to an initial condition. Striking the string with a hammer, as in a piano, is equivalent to giving the string an impulse, so the resulting response is the impulse response (here the string starts at rest so this is the zero state response, or the forced response). In all cases, the different modes are excited and the response is a superposition of the modes. Details of the model are given in the Appendix.

Musical Notes: The table below gives the frequencies of some musical notes.

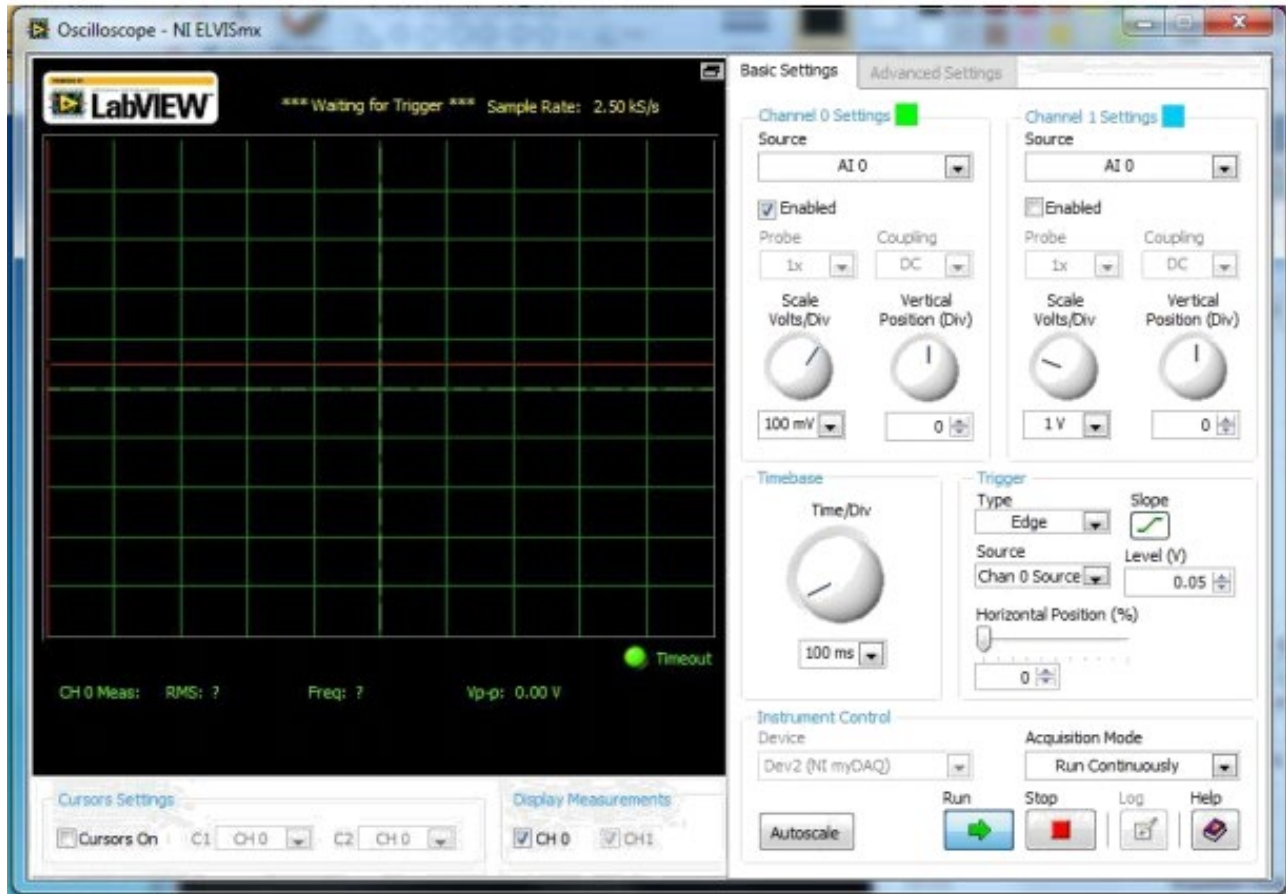
Note	Frequency (Hz)
C	261.6
D	293.7
E	329.6
F	349.0
G	392.0
A	440.0
B	493.9

Breadboards: Breadboards (also known as protoboards) make many connections under the board in order to reduce the number of wires that you have to connect. Typically, groups of five holes are connected. The side rails are lines of groups of 5 that are all connected. See the example figure.



LAB PROCEDURES

1. Plug the myDAQ into the computer USB port first, and then start up the **NI ELVISmx Instrument Launcher** software.
2. Connect the wires from the guitar pickup to the analog input channels, **AI 0+** and **0-** (use the protoboard to help make the connections). The red wire from the pickup should go to **AI 0+** and the black wire from the transducer should go to **AI 0-**.
3. *Set Up the Scope*: Double-click on the **Scope** icon to start up the oscilloscope software and match the settings to the image below, especially,
 - a. Make sure that the **Source** that is selected matches the channel that you are using (AI 0) and make sure that it is enabled.
 - b. Set the **Scale** to 100 mV/div and the vertical position to 0.
 - c. Set the **Timebase** to 100 ms/div, meaning that every vertical line represents 100 ms.
 - d. Set the **Instrument Control Acquisition Mode** to "Run Continuously."
 - e. Set the **Trigger** to match the settings in the image below.
 - f. Click on **Run**.



4. *Zero input response (ZIR) and impulse response:* Move the transducer to a point that is approximately 1/4 along the length of the string (it might already be there). To see a free response due to an initial condition (the ZIR), take a coin and pluck the string near one of its ends and listen to the sound due to the vibrating string. Plucking momentarily moves the string away from its normal resting position and gives it an initial condition. Then try plucking in the middle. Plucking near the end tends to create a brighter sound vs. plucking in the middle, since the deformation of the string in the former case looks like a sawtooth, and in the latter case looks like a triangle, and the sawtooth has richer harmonic content.

It is extremely difficult to repeatedly pluck (or, as in the next part, strike) the string in exactly the same way, so different trials will have different appearances on the **Scope**.

5. To see the impulse response on the scope, take the screwdriver and strike the string sharply with the metal part of the flathead near the end of the string. Hit it sharply enough to hear a tone from the vibrating string. Hitting the string rapidly in this manner is giving it an impulsive input (large force over a very short time span).

Is it not working? If so, check the following:

- Did you plug in the myDAQ before starting the software? Did you click on **Run**?
 - Are the wires loose? The wires need to be inserted securely in the protoboard. Also, the wires that go into the myDAQ interface sometimes come loose and must be re-tightened by loosening the corresponding screw, reinserting the wire, and retightening the screw. Tug gently and/or turn the wire to see how secure it is.
 - Did you connect the transducer to **AI 0+** and **0-** on the myDAQ? Connecting to one of the wrong myDAQ input lines often results in displaying a floating signal that has a lot of 60 Hz component in it.
6. Close the **Scope** window and open the **DSA** window. Match your settings to those shown in the image below, especially note the following settings:
- The **Source** should match the analog input channel from the transducer, **AI 0**
 - **Frequency span** is 4000 Hz and **resolution** 1600
 - **Averaging** is “peak value”
 - **Acquisition Mode** is “Continuous”



7. Noise spectrum: Click on **Run** and examine the frequency spectrum of the signal without any input from you. There may be some ambient **noise** in the measured signal due to vibration of the table inducing a vibration of the string, or some electromagnetic interference from the close proximity of an AC power cords, USB cord, or computer, or even some remnants of digital signal processing induced error such as aliasing. 60 Hz noise, and multiples of 60 Hz, is often present in measurements. **Move** the platform so that the transducer (guitar pickup) is as far away from sources of noise as possible. Look at the frequency spectrum prior to giving the impulsive input. Any peaks in that signal are not due to the impulsive input and should be ignored in your analysis below.
8. Frequency response form an impulsive input: The frequency response can be measured by using the **DSA** to compute the Fourier transform of the impulse response. Use this procedure to compute an approximation of the frequency response:
 - i) Click on **Run**.
 - ii) Click on **Restart** in the Frequency Display settings.
 - iii) Hit the string with an impulse near one of the ends of the string.
 - iv) Click on **Stop** several seconds later to freeze the image on the screen.
 - a. Use the procedure outlined above to obtain a display of the frequency response of the string computed from impulse response data. Use the cursors to determine the frequencies of the fundamental harmonic, second, and third harmonics. To use the cursors, leave C2 cursor at the left margin of the screen (at 0 Hz), move C1 to a peak and read the value of Δf (Hz) below the frequency spectrum plot.

Fundamental _____ Second: _____ Third: _____

Verification 1: Have a lab instructor verify your plot by signing below:

Instructor Signature: _____ **Time:** _____

9. *Modifying the system behavior:* The response of a system depends on the physical characteristics of the system, including the position of the measured output signals and position of the forcing function that gives the input to the system.

a. For the vibrating string system, the fundamental natural frequencies are determined by the physical characteristics of the string as described in the background section: $f \propto T$ and $f \propto 1/L$. The frequency can be tuned by adjusting either the tension, T , or the string and length, L .

i. Turn the knob that adjusts tension to tune the string to a middle C note (± 10 hz is close enough). That is, the fundamental frequency should match that of a middle C given in the background section.

ii. The fret, the upright and movable bar under the string, acts to shorten the length of the string. Calculate the length, with respect to the current length, of the string to obtain an A note.

$L = \underline{\hspace{2cm}}$ (as a fraction of the current length)

Move the fret approximately to that position and push down on the string between the end knob and the fret so that the string is pressed against the fret. Do not push your finger directly on the fret since your finger acts to dampen the signal. Record the frequency response. Verify that the fundamental frequency is that of an A note.

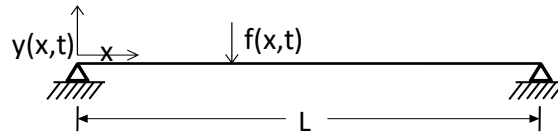
Verification 2: Have a lab instructor verify your plot by signing below:

Instructor Signature: _____ **Time:** _____

iii. (Optional) To see the effect of changing the position of the output measurement device, move the transducer to a node of the second or third modes. Nodes of a modal shape are places where that mode does not contribute to the system's response. Hence, that modal frequency should disappear (or be severely diminished) in the frequency response. Find the frequency response of the system with the transducer placed at a node.

Note that what reaches your ear is a combination of sound waves from the entire string, whereas the pickup is more-or-less "hearing" the string at a single point.

Appendix: Equations of a Vibrating String



L = length of string

T = tension μ = mass/unit length

x = position along the string where a force is applied or an output is defined

y(x,t) = displacement of the string as a function of the position and time

f(x,t) = external force applied to the string

Warning: the notation used here differs slightly from that we have typically used in ECE3084.

Using the quantities defined in the figure, the vibration of the string is modeled as a wave equation with one degree of freedom:

$$T \frac{\partial^2 y}{\partial x^2} - \mu \frac{\partial^2 y}{\partial t^2} = f(x, t)$$

Approximation by ordinary differential equations

The free response of this system has characteristic shapes called “modes of vibration” as seen in the figure in the **Background** section.

The equations describing the modes are given by

$$\varphi_k(x) = \sqrt{\frac{2}{\mu L}} \sin(k\pi x/L)$$

The first mode corresponds to $k=1$, second mode to $k=2$, etc. We can approximate $y(x,t)$ as a superposition of these modes,

$$y(x, t) = \sum_{k=1}^n q_k(t) \varphi_k(x)$$

where $q_k(t)$ represents an amplitude of the mode shape as a function of time. It represents the contribution of that mode shape to the response of the system. This sum is essentially a specialized Fourier series that represents a function in *space*, instead of in time (as in most of the examples we’ve seen), with the Fourier coefficients $q_k(t)$ themselves being functions of time.

If an input force $u(t)$ is applied at a single point $x = x_i$, we can obtain a set of ordinary differential equations that are used to model the system:

$$\ddot{q}_k(t) + 2\xi\omega_k\dot{q}_k(t) + \omega_k^2 q_k(t) = \varphi_k(x_f)u(t) \quad \text{for } k=1,\dots,n$$

where the dots above variables represent *time* derivatives, ξ is called the *damping ratio* (we will discuss that in detail later in the class), and ω_k is the natural frequency of the k^{th} mode in rad/sec:

$$\omega_k = \frac{k\pi}{L} \sqrt{\frac{T}{\mu}}$$

To find the output measured at a single point, $x = x_r$, we can use the series form for $y(x, t)$:

$$y(t) \equiv y(x_r, t) = \sum_{k=1}^n q_k(t) \varphi_k(x_r) \quad \text{for } k=1, n$$

The frequency response of a system can be found by taking the Fourier Transforms of the differential equations and that of the output signal and solving for $Y(j\omega)/U(j\omega)$. In this system, the guitar pickup actually measures velocity of the moving wire, $\dot{y}(t)$, not the displacement, $y(t)$, at the guitar pickup point x , so we actually want $H(j\omega) = Y(j\omega)/U(j\omega)$.

Taking the Fourier transform of both sides of the above ordinary differential equation and using the derivative property of Fourier transform yields (for $k=1, \dots, n$):

$$(j\omega)^2 Q_k(j\omega) + 2\xi\omega_k(j\omega)Q_k(j\omega) + \omega_k^2 Q_k(j\omega) = \varphi_k(x_f)U(j\omega)$$

$$Q_k(j\omega)[(j\omega)^2 + 2\xi\omega_k(j\omega) + \omega_k^2] = \varphi_k(x_f)U(j\omega)$$

$$\frac{Q_k(j\omega)}{U(j\omega)} = \frac{\varphi_k(x_f)}{(j\omega)^2 + 2\xi\omega_k(j\omega) + \omega_k^2} = \frac{\varphi_k(x_f)}{-\omega^2 + j2\xi\omega_k\omega + \omega_k^2}$$

Taking the Fourier transform of both sides of the series form for $y(x, t)$ and applying the derivative property of Fourier transforms once more yields:

$$\dot{Y}(j\omega) = j\omega \sum_{k=1}^n Q_k(j\omega) \varphi_k(x_r)$$

From our last two equations, we can compute the frequency response function

$$H(j\omega) = \frac{\dot{Y}(j\omega)}{U(j\omega)} = j\omega \sum_{k=1}^n \frac{\varphi_k(x_r) \varphi_k(x_f)}{-\omega^2 + j2\xi\omega_k\omega + \omega_k^2}$$